IHEP 99–12
hep-ph/9903344

Vector-like family extension of the standard model and the light quark masses and mixings

Yury F. Pirogov[1]
Oleg V. Zenin

Institute for High Energy Physics,
Protvino, RU-142284 Moscow Region, Russia

Moscow Institute of Physics and Technology,
Dolgoprudny, Moscow Region, Russia

Abstract

The standard model extended with the pairs of the vector-like families is studied. The model independent analysis for an arbitrary case and an explicit realization for the case with one pair of the heavy vector-like families are considered. The mixing matrices of the light quarks for the left- and right-chiral charged currents, as well as those for the flavour changing neutral currents, both the Z and Higgs mediated, are found.

[1] E-mail address: pirogov@mx.ihep.su

1. Introduction Are there extra families in the standard model (SM) or not this is the question. A recent two-loop renormalization group analysis [1] of the SM shows that subject to the precision experiment restriction on the Higgs mass, $M_H \le 200$ GeV at 95% C.L. [2], the forth chiral family, if alone, is excluded.[2] In fact, it does not depend on whether this extra family has the normal chiral structure or the mirror one. But as it is noted in the Ref. [1], a pair of the opposite chirality families with the relatively low Yukawa couplings evades the SM self-consistency restrictions and could still exist. In order to conform with observations these extra families, which otherwise can be considered as the vectorial ones, should get large direct masses and to drop out of the light particle spectrum of the SM in the decoupling limit. Nevertheless, at the moderate masses, say of order 1 TeV or so, such families could lead to observable corrections to the SM interactions through mixing with the light fermions.

The various vector-like fermions are generic in many extensions of the SM like the superstring and grand unified theories, composite models, etc. Many issues concerning those fermions, both the electroweak doublets and singlets, the latter ones of the up and down types, were considered in the literature [4], [5]. On the other hand there are numerous studies of the $n > 3$ chiral family extensions of the SM [6], [7]. Some topics concerning the SM extensions with the vector-like families are studied in Ref. [8]. But the problem of the SM quark masses and mixings in the presence of the extra vector-like families have not yet found its full model independent consideration, and it is studied in the current paper. We present both the model independent analysis for the general case and an explicit realization for the case with a pair of the heavy vector-like families.

2. Arbitrary number of the vector-like families The most general content of the SM families consisting of the $\mathrm{SU(2)_W} \times \mathrm{U(1)_Y}$ doublets and singlets in the chiral notations is $nQ_L + mQ'_R$, where $Q_L = (\hat{q}_L, \hat{u}^c_L, \hat{d}^c_L)$ and $Q_R = (\hat{q}'_R, \hat{u}'^c_R, \hat{d}'^c_R)$. The symbols with the hat sign designate quarks in the

[2]More conservative restrictions $m_H \le 262$ GeV or $M_H \le 300$ GeV at 95% C.L., respectively, from the first and second papers of Ref. [3] though render this conclusion somewhat less reliable, nevertheless do not invalidate it.

symmetry/electroweak basis where, by definition, the SM symmetry structure is well stated. Here $n \geq 3$ is the number of chiral families, similar in their chiral and quantum number structure to three ordinary families of the minimal SM. $m \geq 0$ means the number of the mirror conjugate families with the normal quantum numbers, or in other terms, the charge conjugate families with the normal chiral structure. In the more traditional left-right notations, one should substitute: $Q_L \to (\hat{q}_L, \hat{u}_R, \hat{d}_R)$ and $Q_R \to (\hat{q}'_R, \hat{u}'_L, \hat{d}'_L)$.

In general, quarks gain masses from two different physical mechanisms: that of the SM Yukawa interactions and that of a New Physics resulting in the SM invariant direct mass terms. Being chirally unprotected the latter ones should naturally be characterized by a high mass scale M, $M \gg v$, with v being the SM Higgs vacuum expectation value. In the symmetry basis the kinetic, Yukawa and direct mass Lagrangian has the following most general form:

$$\begin{aligned}
\mathcal{L} \;=\; & \; i\overline{\hat{q}_L}\not{D}\hat{q}_L + i\overline{\hat{u}_R}\not{D}\hat{u}_R + i\overline{\hat{d}_R}\not{D}\hat{d}_R \\
& + i\overline{\hat{q}'_R}\not{D}\hat{q}'_R + i\overline{\hat{u}'_L}\not{D}\hat{u}'_L + i\overline{\hat{d}'_L}\not{D}\hat{d}'_L \\
& - \Big(\overline{\hat{q}_L}Y^u\hat{u}_R\phi^c + \overline{\hat{q}_L}Y^d\hat{d}_R\phi + \overline{\hat{u}'_L}Y^{u'}\hat{q}'_R\phi^{c\dagger} + \overline{\hat{d}'_L}Y^{d'}\hat{q}'_R\phi^\dagger + \text{h.c.}\Big) \\
& - \Big(\overline{\hat{q}_L}M\hat{q}'_R + \overline{\hat{u}'_L}M^{u'}\hat{u}_R + \overline{\hat{d}'_L}M^{d'}\hat{d}_R + \text{h.c.}\Big)\,, \qquad (1)
\end{aligned}$$

where $\not{D} \equiv \gamma^\mu D_\mu$ is the SM covariant derivative and ϕ is the Higgs doublet and ϕ^c is the garged conjugate one. In Eq. (1), Y and Y' are, respectively, the square $n \times n$ and $m \times m$ Yukawa matrices; M and M' are, respectively, the rectangular $n \times m$ and $m \times n$ direct mass matrices.

We generalize the parameter counting for the chiral families of Ref. [7] to the case with the extra vector-like families (VLF's). It goes as is shown in Table 1. Here G is the global symmetry of the kinetic part of the Lagrangian (1). It is broken explicitly by the mass terms, only the residual symmetry $H = U(1)$ of the baryon number being left in the general case we consider. Hence, the transformations of G/H can be used to absorb the spurious parameters in Eq. (1) leaving only the physical set $\mathcal{M}_{phys}$ of them. Of the physical moduli, the $2(n+m)$ ones are the physical masses, the rest being mixing angles. The last two lines in Table 1 present the physical parameters for the minimal SM and for its extension with a pair of the normal and mirror families. This case will be considered in detail further on.

Let us now redefine collectively quarks in the symmetry basis as $\hat{\kappa}_\chi = \hat{u}_\chi$,

Table 1 Parameter counting in the symmetry/electroweak basis.

Couplings and symmetries	Moduli	Phases
$Y^u, Y^d, Y^{u'}, Y^{d'}$, $M, M^{u'}, M^{d'}$	$2\ (n^2+m^2)$ $+3\ nm$	$2\ (n^2+m^2)$ $+3\ nm$
$G = U(n)^3 \times U(m)^3$	$-\frac{3}{2}[n(n-1)+m(m-1)]$	$-\frac{3}{2}[n(n+1)+m(m+1)]$
$H = U(1)$	0	1
$\mathcal{M}_{phys}(n,m)$	$\frac{1}{2}(n+m)(n+m-1)$ $+2\ nm +\ 2\ (n+m)$	$\frac{1}{2}(n+m-2)(n+m-1)$ $+2\ nm$
$\mathcal{M}^{SM}_{phys}(3,0)$	$6+3=9$	1
$\mathcal{M}_{phys}(4,1)$	$10+18=28$	14

$\hat{d}_\chi$ and these in the mass basis, i.e. the quark eigenstates with $\mathcal{M}_{phys}$ being diagonal, as $\kappa_\chi = u_\chi,\ d_\chi$ ($\chi = L,\ R$). The bases are related by the unitary $(n+m)\times(n+m)$ transformations

$$\hat{\kappa}_{\chi A} = (U^\kappa_\chi)^F_A\, \kappa_{\chi F}\ , \tag{2}$$

with the ensuing bi-unitary mass diagonalization

$$U^{\kappa\dagger}_L \mathcal{M}^k U^\kappa_R = \mathcal{M}^\kappa_{diag} = \mathrm{diag}\,(\overline{m}^\kappa{}_f, \overline{M}^\kappa{}_4, \ldots, \overline{M}^\kappa{}_{n+m})\ . \tag{3}$$

In equations above, the indices $A = A_L, A_R$; $A_L = 1, \ldots, n$; $A_R = n+1, \ldots, n+m$ are those in the symmetry basis, and $F = f, 4, \ldots, n+m$; $f = 1, 2, 3$ are indices in the mass basis. It is assumed that $\overline{m}^\kappa{}_f \ll \overline{M}^\kappa{}_4, \ldots, \overline{M}^\kappa{}_{n+m}$.

The matrices U^κ_χ satisfy the unitarity relations

$$U^\kappa_\chi\, U^{\kappa\dagger}_\chi = I \tag{4}$$

and

$$U^{\kappa\dagger}_\chi I_L U^\kappa_\chi + U^{\kappa\dagger}_\chi I_R U^\kappa_\chi = I\ , \tag{5}$$

were I_L, I_R are the projectors onto the normal and mirror subspaces in the symmetry basis:

$$\begin{aligned} I_L &= \mathrm{diag}\,(\underbrace{1, \ldots, 1}_{n}; \underbrace{0, \ldots, 0}_{m})\ , \\ I_R &= \mathrm{diag}\,(\underbrace{0, \ldots, 0}_{n}; \underbrace{1, \ldots, 1}_{m}) \end{aligned} \tag{6}$$

with $I_L + I_R = I$ and $I_\chi^2 = I_\chi$. Let us also introduce their transforming to the mass basis

$$X_\chi^\kappa = U_\chi^{\kappa\dagger} I_\chi U_\chi^\kappa \ . \tag{7}$$

($\kappa = u$, d and $\chi = L$, R). Clearly, X_χ^κ are Hermitian and satisfy the projector condition: $X_\chi^{\kappa 2} = X_\chi^\kappa$ (but note that $X_L^\kappa + X_R^\kappa \neq I$ in the notations adopted).

Now, the charged current Lagrangian is

$$-\mathcal{L}_W = \frac{g}{\sqrt{2}} W_\mu^+ \sum_\chi \overline{u_\chi} \gamma^\mu V_\chi d_\chi + \text{h.c.} \tag{8}$$

and the neutral current one is

$$-\mathcal{L}_Z = \frac{g}{c} Z_\mu \sum_{\kappa,\chi} \overline{\kappa_\chi} \gamma^\mu N_\chi^\kappa \, \kappa_\chi \ , \tag{9}$$

where $c \equiv \cos\theta_W$, with θ_W being the Weinberg mixing angle. The corresponding quark mixing matrices for the charged currents are:

$$V_\chi = U_\chi^{u\dagger} I_\chi U_\chi^d \ , \tag{10}$$

and for the neutral currents with the operator $T_3 - s^2 Q$:

$$N_\chi^\kappa = T_3^\kappa X_\chi^\kappa - s^2 Q_\chi^\kappa \ . \tag{11}$$

Here T_3^κ is the 3rd component of the electroweak isospin for $\kappa = u, d$ and $Q_{L,R}^\kappa \equiv Q^\kappa I$, with Q^κ being the corresponding electric charge; $s \equiv \sin\theta_W$.

The charged current mixing matrices V_L and V_R play the role of the generalized CKM matrices. But contrary to the minimal SM case, they as well as the neutral current mixing matrices N_χ^κ are non-unitary. Namely, one gets by the unitarity relations (4)

$$\begin{aligned} V_\chi V_\chi^\dagger &= X_\chi^u \ , \\ V_\chi^\dagger V_\chi &= X_\chi^d \ , \end{aligned} \tag{12}$$

where X_χ^κ ($X_\chi^\kappa \neq I$ in general) are given by Eq. (7).

It is seen that the neutral current matrices N_χ^κ are not independent of the charged current ones V_χ. In fact, one can convince oneself that V_χ and the diagonal mass matrices $\mathcal{M}_{diag}^\kappa$ suffice to parametrize all the fermion interactions in a general class of the SM extensions by means of the arbitrary

numbers of the vector-like isodoublets and isosinglets [5]. Indeed, in the case at hand using the unitarity relations (5) one gets for the Yukawa Lagrangian in the unitary gauge

$$\begin{aligned} -\mathcal{L}_Y &= \frac{H}{v}\sum_{\kappa}\overline{\kappa_L}\Big(X_L^\kappa \mathcal{M}_{diag}^\kappa - 2X_L^\kappa \mathcal{M}_{diag}^\kappa X_R^\kappa + \mathcal{M}_{diag}^\kappa X_R^\kappa\Big)\kappa_R \\ &\quad + \sum_{\kappa}\overline{\kappa_L}\mathcal{M}_{diag}^\kappa \kappa_R + \text{h.c.} \ , \end{aligned} \tag{13}$$

H being the physical Higgs boson. It follows from the above expression and Eqs. (9), (11) that all the flavour changing neutral currents are induced entirely by the lack of unitarity of the charged current mixing matrices V_χ. In the case with only the normal families ($X_L^\kappa = I$, $X_R^\kappa = 0$) the usual SM expressions for $\mathcal{L}_W$, $\mathcal{L}_Z$ and $\mathcal{L}_Y$ are recovered, the two latter ones being flavour conserving.

We propose the following prescription for the model independent parametrization of the V_χ. The problem is that they are non-unitary and thus are difficult to parametrize directly. So, the idea is to express them in terms of a set of the auxiliary unitary matrices. First of all, note that in absence of any restrictions on the Lagrangian the unitary matrices U_χ^κ in Eq. (2) would be arbitrary. Now, an arbitrary $(n+m)\times(n+m)$ unitary matrix U can always be uniquely decomposed as $U = U|_{n\times n}\, U|_{m\times m}\, U|_{n\times m}$. Here $U|_{n\times n}$ is a unitary matrix in the $n\times n$ subspace. It is built of the n^2 generators. Similarly, $U|_{m\times m}$ is the restriction of U onto the $m\times m$ subspace, and it is built of the m^2 generators. And finally, $U|_{n\times m}$ means a unitary $(n+m)\times(n+m)$ matrix built of the $2nm$ generators which mix the two subspaces.

Now, by means of the symmetry basis transformations G of the Table 1 one can always put, without loss of generality, the matrices U_χ^κ to the form

$$\begin{aligned} U_L^u &= U_L^u|_{n\times m} \ , \\ U_R^u &= U_R^u|_{n\times m} \ , \\ U_L^d &= U_L^d|_{n\times n}\, U_L^d|_{n\times m} \ , \\ U_R^d &= U_R^d|_{m\times m}\, U_R^d|_{n\times m} \ . \end{aligned} \tag{14}$$

This representation includes six auxiliary unitary matrices. Clearly, they depend on the $[n(n-1)/2 + m(m-1)/2 + 4mn]$ moduli and $[n(n+1)/2 + m(m+1)/2 + 4mn]$ phases, and these numbers are redundant. But the nm

moduli and the same number of phases can be eliminated through the $n \times m$ matrix constraint

$$I_L U_L^u \mathcal{M}_{diag}^u U_R^{u\dagger} I_R = I_L U_L^d \mathcal{M}_{diag}^d U_R^{d\dagger} I_R \ . \tag{15}$$

The latter follows from the equality of the direct mass matrices M in Eq. (1) for the up and down quarks, and it includes additionally the $2(n+m)$ independent moduli which enter $\mathcal{M}_{diag}^u$ and $\mathcal{M}_{diag}^d$. By means of the Eq. (15) one can express, e.g., one of the $U_\chi^\kappa|_{n\times m}$ in terms of all other matrices. And finally, the $2(n+m)-1$ phases can be removed via the residual phase redefinition for the quarks in the mass basis. Putting all together, one can easily verify that the total number of the independent parameters is precisely as expected from the Table 1.

Having parametrized the auxiliary unitary matrices, one gets for the V_χ

$$\begin{aligned} V_L &= U_L^{u\dagger}|_{n\times m}\ I_L\ U_L^d|_{n\times n}\ U_L^d|_{n\times m}\ , \\ V_R &= U_R^{u\dagger}|_{n\times m}\ I_R\ U_R^d|_{m\times m}\ U_R^d|_{n\times m} \end{aligned} \tag{16}$$

and for the X_χ^κ

$$X_\chi^\kappa = U_\chi^{\kappa\dagger}|_{n\times m}\ I_\chi\ U_\chi^\kappa|_{n\times m}\ . \tag{17}$$

When eliminating the $2(n+m)-1$ redundant phases one can always take such a choice as to render the diagonal and above-the-diagonal elements of the V_L (or V_R) to be real and positive.

This gives the principal solution to the problem. When there are only the normal families ($m=0$) the usual parametrization in terms of just one unitary matrix $U_L^d|_{n\times n}$ is readily recovered. For the case with a pair of VLF's ($n=4$, $m=1$) we got also the explicit expressions of all the relevant quantities in terms of a minimal common set of the independent arguments parametrizing the mass matrices (see below).

3. A pair of the heavy vector-like families The mass/flavour basis quantities, $\mathcal{M}_{diag}^{u,d}$ and $V_{L,R}$, are phenomenological by their very nature. They reflect an obscure mixture of contributions of the quite different physical origin. In particular, they shed no light on the mixing magnitudes. On the contrary, the parameters in the symmetry basis, i.e. Yukawa couplings and the direct mass terms M and $M^{u\prime}$, $M^{d^\prime}$ have the straightforward theoretical meaning. So, we express the former ones in terms of the latter ones. This

permits us to expand upon the idea of the relative magnitude of the various mixing elements in terms of the small quantity v/M.

The asymptotic freedom requirement for the $\mathrm{SU(2)_W}$ electroweak interactions results in the restriction that the total number of the electroweak doublets should not exceed 21, and thus the total number of the families is $(n+m) \leq 5$. Hence the maximum number of the extra VLF's allowed by the asymptotic freedom is two, the case we stick to in what follows.

Using here the global symmetries G of the Table 1 one can bring, without loss of generality, the quark mass matrices in the symmetry basis to the following canonical form

$$\mathcal{M}^\kappa = \begin{pmatrix} m^{\kappa g}_{\ f} & \mu^{\kappa\prime}_{\ f} & 0 \\ \mu^{\kappa g} & m^\kappa{}_4 & M \\ 0 & M^{\kappa\prime} & m^\kappa{}_5 \end{pmatrix} , \tag{18}$$

where M, $M^{\kappa\prime}$ are the real scalars and $\mu^{\kappa f}$, $\mu^{\kappa\prime}{}_f$, $m^\kappa{}_4$, $m^\kappa{}_5$ are in general complex. Here the lower case characters generically mean the masses of the Yukawa origin ($\sim Yv$). Let us remind that M in Eq. (18) is common for both $\mathcal{M}^u$ and $\mathcal{M}^d$. The three-dimensional matrices m^κ are Hermitian and positive definite, and one of them, e.g. m^u, can always be chosen diagonal. Under such a choice one can simplify further:

$$\mathcal{M}^\kappa_0 = U^{\kappa\dagger}_0 \mathcal{M}^\kappa U^\kappa_0 , \tag{19}$$

where

$$\mathcal{M}^\kappa_0 = \begin{pmatrix} m^\kappa{}_1 & 0 & 0 & \mu^{\kappa\prime}_1 & 0 \\ 0 & m^\kappa{}_2 & 0 & \mu^{\kappa\prime}_2 & 0 \\ 0 & 0 & m^\kappa{}_3 & \mu^{\kappa\prime}_3 & 0 \\ \mu^{\kappa 1} & \mu^{\kappa 2} & \mu^{\kappa 3} & m^\kappa{}_4 & M \\ 0 & 0 & 0 & M^{\kappa\prime} & m^\kappa{}_5 \end{pmatrix} \tag{20}$$

with a redefinition of $\mu^{\kappa f}$ and $\mu^{\kappa\prime}{}_f$, and with the diagonal elements $m^\kappa{}_f$ being real and positive. The corresponding unitary U^κ_0 are given by

$$\begin{aligned} U^u_0 &= I , \\ U^d_0 &= \begin{pmatrix} V_C & 0 \\ 0 & I_2 \end{pmatrix} , \end{aligned} \tag{21}$$

V_C being the 3×3 CKM matrix and I_2 the 2×2 identity matrix. The mass matrices of Eq. (20) possess the residual symmetry $U(1)^6$ which is reduced to $U(1)^5$ by the baryon number conservation. So, one can use phase redefinitions for two of the light d quarks which leave just one complex phase in V_C in accordance with the decoupling limit requirement.

It is seen from Eqs. (20) and (21) that in this parametrization the total number of the physical moduli is $10+15+3 = 28$ as it should be according to the Table 1. What concerns the phases, their number is in general $16+1 = 17$, i.e. three of them are spurious and can be removed. E.g., by means of the residual phase redefinition for the three light u quarks one can make μ^{uf} or ${\mu^{u}}'_f$ to be real, or put some other three relations on their phases. This exhausts the freedom of the phase redefinitions, leaving only the physical parameters.

Solving the characteristic equations $\det(\mathcal{M}_0^\kappa \mathcal{M}_0^{\kappa\dagger} - \overline{m}^{\kappa 2} I) = 0$ one gets for the light quark physical masses in the first order (i.e. up to the relative corrections $\mathcal{O}(v^2/M^2)$ to the leading order):

$$\overline{m}_f^2 = m_f^2\Big(1 - \big(\frac{|\mu^f|^2}{M^2} + \frac{|\mu'_f|^2}{M'^2}\big)\Big) + \frac{m_f}{MM'}(m_5 \mu^f \mu'_f + \text{h.c.}) \qquad (22)$$

with the superscripts $\kappa = u,\, d$ being suppressed. Here it is supposed that $M \sim M'$ but $M \neq M'$ in general. It is seen that corrections to m_f^2 are proportional to m_f themselves, i.e. the light quarks are chirally protected. This property drastically reduces the otherwise dangerous corrections to the masses of the lightest u and d quarks at the moderate M. On the other hand, it means that the masses of the lightest quarks can not entirely be induced by an admixture of the vector-like families: if $m_f = 0$ then $\overline{m}_f = 0$, too.

Once the physical masses are known, one can obtain the matrices U_L^κ and U_R^κ of the bi-unitary transformation (3). With account for Eq. (10) one gets hereof for the light quark mixing matrix V_L

$$V_L{}^g_f = V_C{}^g_f\Big(1 - \frac{1}{2M^2}({n^u}^f_f + {n^d}^g_g)\Big) - \frac{1}{M^2}\sum({p^u}^{f*}_h V_C{}^g_h + V_C{}^h_f {p^d}^g_h) \qquad (23)$$

and similarly for V_R

$$V_R{}^g_f = \frac{1}{M^{u\prime} M^{d\prime}} {p^{u\prime}}^{f*}_5 {p^{d\prime}}^g_5\,, \qquad (24)$$

where

$$
\begin{aligned}
p_g^f &= \frac{\mu^f(m_f^2-|m_5|^2)(m_f\mu^{f*}\mu'_g - m_g\mu^{g*}\mu'_f) + k_f(m_f\mu'_g - \frac{m_g}{m_f}\frac{M'}{M}\mu^{g*}m_5^*)}{(m_g^2-m_f^2)(m_f\mu'_f - \frac{M'}{M}m_5^*\mu^{f*})} ,\\
p_5^f &= \frac{\frac{M'}{M}(k_f+m_f^2|\mu^f|^2) - m_f m_5\mu^f\mu'_f}{m_f(m_f\mu'_f - \frac{M'}{M}m_5^*\mu^{f*})} ,\\
n_f^f &= \left|\frac{\frac{M'}{M}(k_f+m_f^2|\mu^f|^2) - m_f m_5\mu^f\mu'_f}{m_f(m_f\mu'_f - \frac{M'}{M}m_5^*\mu^{f*})}\right|^2
\end{aligned}
\tag{25}
$$

with $k_f = M^2(\overline{m}_f^2 - m_f^2)$. The p', n' are obtained from p, n, respectively, by substituting $\mu^f \leftrightarrow {\mu'_f}^*$, $m_4 \leftrightarrow m_4^*$, $m_5 \leftrightarrow m_5^*$, $M \leftrightarrow M'$. All these auxiliary parameters are in general of order $\mathcal{O}(M^0)$.

The charged current Lagrangian $\mathcal{L}_W$ is given by Eq. (8). The Z mediated neutral current Lagrangian $\mathcal{L}_Z$ is as given by Eqs. (9), (11) with

$$
{X_L}_f^g = \delta_f^g - \frac{1}{M^2}p_5^{f*}p_5^g \tag{26}
$$

and

$$
{X_R}_f^g = \frac{1}{M'^2}{p'}_5^{f*}{p'}_5^g \ . \tag{27}
$$

The neutral scalar current Lagrangian takes the general form

$$
-\mathcal{L}_H = \frac{H}{v}\sum_\kappa \overline{\kappa_L}\, U_L^{\kappa\dagger}(\mathcal{M}^\kappa - \mathcal{M}_{dir}^\kappa)U_R^\kappa\,\kappa_R + \text{h.c.} \tag{28}
$$

with the direct mass matrices

$$
\mathcal{M}_{dir}^\kappa = \begin{pmatrix} O_3 & 0 & 0 \\ 0 & 0 & M \\ 0 & M^{\kappa\prime} & 0 \end{pmatrix} , \tag{29}
$$

where O_3 is the 3×3 zero matrix. As a consequence of the substraction of the direct mass terms, the total mass and Yukawa matrices are not diagonalizable simultaneously in the same basis. In the mass basis, the Higgs interaction Lagrangian is non-diagonal

$$
-\mathcal{L}_H \;=\; \frac{H}{v}\sum_\kappa \overline{\kappa_L}\;\mathcal{H}^\kappa\kappa_R + \text{h.c.} \ , \tag{30}
$$

with the light quark mixing matrix (indices $\kappa = u$, d being omitted)

$$\mathcal{H}_f^g = \overline{m}_f \delta_f^g - \frac{1}{MM'}\Big(p_4^{f*} p'^g_5 + p_5^{f*} p'^g_4\Big) \ , \tag{31}$$

where

$$p_4^f = -k_f \ \frac{\Big(k_f + |\mu^f|^2 (m_f^2 - |m_5|^2)\Big)\Big(\frac{M'}{M} m_5^* + \frac{1}{k_f} m_f \mu^f \mu'_f (m_f^2 - |m_5|^2)\Big)}{m_f (m_f \mu'_f - \frac{M'}{M} m_5^* \mu_f^*)} \tag{32}$$

with p'^f_4 being obtained from it by the usual substitutions.

One should stress that for the light quarks all the off-diagonal components of the Lagrangian $\mathcal{L}_W$ (beyond that of the minimal SM), as well as those of the $\mathcal{L}_Z$ and $\mathcal{L}_H$ are suppressed by the ratio v^2/M^2, and it does not depend on the details of the mass matrices.

4. Conclusions We have shown that the mere addition of a pair of the VLF's drastically changes all the characteristic features of the minimal SM. First of all, the generalized CKM matrix for the left-handed charged currents ceases to be unitary. Moreover, this non-unitarity takes place in the whole flavour space but not only in the light quark sector which would occur for adding only the normal families. Further, there appear the right-handed charged currents, the flavour changing neutral currents, both the vector and scalar ones, all with the non-unitary mixing matrices and with a number of CP violating phases.

Due to decoupling under the large direct mass terms M, the extended SM definitely does not contradict to experiment in the limit $M \gg v$. But at the moderate $M > v$, the addition of a pair of the VLF's would make the model phenomenology, especially that of the flavour and CP violation, extremely reach. It is to be seen what is the real experimentally allowed region in the parameter space for the VLF's and what are the possibilities to observe their effects in the future experiments. We hope that our paper will stimulate further study in this direction.

References

[1] Yu.F. Pirogov and O.V. Zenin, IHEP preprint IHEP 98–50 (1998), to be publ. in Eur. Phys. J. C, 1999, hep-ph/9808396; Report presented at the Int. Seminar "Quarks '98", Suzdal, May 17-24, 1998, hep-ph/9808414.

[2] E. Tournefier (ALEPH Collab.), Report presented at the Int. Seminar "Quarks '98", Suzdal, May 17-24, 1998.

[3] F. Teubert, hep-ph/9811414; G. D'Agostini and G. Degrassi, hep-ph/9902226.

[4] G.C. Branco and L. Lavoura, Nucl. Phys B278 (1986) 738; Phys. Lett. B208 (1988) 123; B. Mukhopadhyaya and S. Nandi, Phys. Rev. Lett. 66 (1991) 285; Phys. Rev. D46 (1992) 5098; W.S. How, Phys. Rev. Lett. 69 (1992) 3587; T.P. Cheng and L.F. Li, Phys. Rev. D45 (1992) 1708; G.C. Branco, T. Morozumi, P. Parada and M.N. Rebelo, *ibid* D48 (1993) 1167; G.C. Branco, P. Parada and M.N. Rebelo, *ibid* D52 (1995) 4217, hep-ph/9501347; G. Bhattacharyya, G.C. Branco and W.S. How, Phys. Rev. D54 (1996) 2114, hep-ph/9512239; L.T. Handoco and T. Morozumi, Mod. Phys. Lett. A10 (1995) 309, hep-ph/9409240; E. Ma, Phys. Rev. D53 (1996) 2276, hep-ph/9510289; T. Yoshikawa, Prog. Theor. Phys. 96 (1996) 269, hep-ph/9512251.

[5] L. Lavoura and J.P. Silva, Phys. Rev. D47 (1993) 1117.

[6] J. Shechter and J.W.F. Valle, Phys. Rev. D21 (1980) 309; D22 (1980) 2227; R. Mignani, Lett. Nuovo Cimento 28 (1980) 529; C. Jarlskog, Phys. Rev. Lett. 55 (1985) 1039; Z. Phys. C29 (1985) 491; Phys. Rev. D35 (1987) 1685; M. Gronau, R. Johnson and J. Shechter, *ibid* D32 (1985) 3062; M. Gronau, A. Kfir and R. Loewy, Phys. Rev. Lett. 56 (1986) 1538; J. Bernabeu, G.C. Branco and M. Gronau, Phys. Lett. B169 (1986) 243; H. Harari and M. Leurer, *ibid* B181 (1986) 123; H. Frietzsch and J. Plankl, Phys. Rev. D35 (1987) 1732; J.F. Nieves and P.B. Bal, *ibid* D36 (1987) 315; J.D. Bjorken and I. Dunitz, *ibid* D36 (1987) 2109.

[7] A. Santamaria, Phys. Lett. B305 (1993) 90, hep-ph/9302301.

[8] K. Fujikawa, Prog. Theor. Phys. 92 (1994) 1149; hep-ph/9604358; R.K. Babu, J. Pati and H. Stremnitzer, Phys. Rev. D51 (1995) 2451, hep-ph/9409381; H. Zheng, hep-ph/9602340.